VI Računske vežbe

Buka u zatvorenom prostoru

1. U reverberacionu prostoriju dimenzija 8×5×3 m i vremena reverberacije 3,5 s, uneto je 15 m² apsorpcionog materijala nepoznatog koeficijenta apsorpcije. Vreme reverberacije u novim uslovima ima vrednost 1,25 s. Izračunati koeficijent apsorpcije unetog materijala. (1. knjiga)

$a = 8\,m$

$b = 5\,m$

$c = 3\,m$

$T_{R0} = 3,5\,s \quad T_{R0} > 0,85 \Rightarrow$ Sabinov obrazac

$S_1 = 15\,m^2$

$T_{R1} = 1,25\,s \quad T_{R1} > 0,85 \Rightarrow$ Sabinov obrazac

$\alpha_1 = ?$

$S_0 = 2(ab + ac + bc) = 2 \cdot (8 \cdot 5 + 8 \cdot 3 + 5 \cdot 3) = 2 \cdot (40 + 24 + 15) = 2 \cdot 79 = 158\,m^2$

$\boxed{S_0 = 158\,m^2}$

$\boxed{T_{R0} = 0,162 \frac{V}{A_0}}$

$V = abc \Rightarrow V = 8 \cdot 5 \cdot 3\,m^3 = 120\,m^3 \Rightarrow \boxed{V = 120\,m^3}$

$A_0 = S_0 \bar{\alpha}_0$

$T_{R0} = 0,162 \frac{V}{S_0 \bar{\alpha}_0} \Rightarrow \boxed{\bar{\alpha}_0 = 0,162 \frac{V}{S_0 T_{R0}}}$

$\bar{\alpha}_0 = 0,162 \cdot \frac{120}{158 \cdot 3,5} = \boxed{0,035}$

$$T_{R1} = 0{,}162 \frac{V}{A_1}$$

$$A_1 = \sum_i S_i \alpha_i = (S_0 - S_1)\bar{\alpha}_0 + S_1 \bar{\alpha}_1$$

$$T_{R1} = 0{,}162 \frac{V}{(S_0 - S_1)\bar{\alpha}_0 + S_1 \bar{\alpha}_1}$$

$$0{,}162 V = T_{R1}\left[(S_0 - S_1)\bar{\alpha}_0 + S_1 \bar{\alpha}_1\right]$$

$$\frac{0{,}162 V}{T_{R1}} = (S_0 - S_1)\bar{\alpha}_0 + S_1 \bar{\alpha}_1$$

$$\bar{\alpha}_1 = \frac{\frac{0{,}162 V}{T_{R1}} - (S_0 - S_1)\bar{\alpha}_0}{S_1}$$

$$\bar{\alpha}_1 = \frac{0{,}162 V}{S_1 T_{R1}} - \frac{S_0 - S_1}{S_1} \cdot 0{,}162 \frac{V}{S_0 T_{R0}}$$

$$\boxed{\bar{\alpha}_1 = \frac{0{,}162 V}{S_1}\left(\frac{1}{T_{R1}} - \frac{S_0 - S_1}{S_0 T_{R0}}\right)}$$

$$\bar{\alpha}_1 = \frac{0{,}162 \cdot 120}{15}\left(\frac{1}{1{,}25} - \frac{158 - 15}{158 \cdot 3{,}5}\right) = 8 \cdot 0{,}162\left(0{,}8 - \frac{143}{553}\right)$$

$$\boxed{\bar{\alpha}_1 = 0{,}7}$$

(2.) Odrediti zvučnu snagu izvora u prostoriji dimenzija 10 x 8 x 5 m, sa srednjim koeficijentom refleksije 0,9, ako je u prostoriji formiran nivo buke od 80 dB. (2. knjiga)

$a = 10\,m$

$b = 8\,m$

$c = 5\,m$

$\bar{r} = 0,9$

$L = 80\,dB$

$P_a = ?$

$S = 2(ab + ac + bc) = 2 \cdot (10 \cdot 8 + 10 \cdot 5 + 8 \cdot 5) = 2 \cdot (80 + 50 + 40) = 2 \cdot 170\,m^2$

$\boxed{S = 340\,m^2}$

$\bar{\alpha} + \bar{r} = 1 \Rightarrow \bar{\alpha} = 1 - \bar{r} = 1 - 0,9 = 0,1 \Rightarrow \boxed{\bar{\alpha} = 0,1}$

$\bar{\alpha} = 0,1 < 0,3 \Rightarrow$ ispunjeni uslovi homogenosti i difuznosti zvučnog polja u prostoriji

$A = \sum_i S_i \alpha_i = S\bar{\alpha} = 340\,m^2 \cdot 0,1 = 34\,m^2 \Rightarrow \boxed{A = 34\,m^2}$

$L = 10 \log \frac{I}{I_0}, \quad I_0 = 10^{-12} \frac{W}{m^2}$

$\frac{L}{10} = \log \frac{I}{I_0} \Rightarrow \frac{I}{I_0} = 10^{\frac{L}{10}} \Rightarrow \boxed{I = I_0 \cdot 10^{\frac{L}{10}}}$

$I = 10^{-12} \frac{W}{m^2} \cdot 10^{\frac{80}{10}} = 10^{-12} \cdot 10^{8} \frac{W}{m^2} = 10^{-4} \frac{W}{m^2} \Rightarrow \boxed{I = 10^{-4} \frac{W}{m^2}}$

$I = \frac{4P_a}{A} \Rightarrow \boxed{P_a = \frac{I \cdot A}{4}}$

$P_a = \frac{10^{-4} \frac{W}{m^2} \cdot 34\,m^2}{4} = \frac{34}{4} \cdot 10^{-4}\,W \Rightarrow \boxed{P_a = 8,5 \cdot 10^{-4}\,W}$

(3.) Tačkasti izvor buke zvučne snage 5W instaliran je u prostoriji zapremine $100\,m^3$, sa vremenom reverberacije 4s. Ako se izvor buke postavi na otvorenom prostoru, odrediti na kom rastojanju od izvora bi intenzitet zvuka bio isti kao i u prostoriji. Za tu situaciju odrediti nivo zvučnog pritiska. (3. način)

$P_a = 5W$

$V = 100\,m^3$

$T_R = 4s$, $T_R > 0,8s$ => Sabinov obrazac

$I_0 = 10^{-12}\,\frac{W}{m^2}$; $p_0 = 20\mu Pa = 2\cdot 10^{-5}\,Pa$

$r = ?$

$L = ?$

$T_R = 0,162\,\frac{V}{A}$ => $\boxed{A = 0,162\,\frac{V}{T_R}}$

$I = \frac{4P_a}{A} = \frac{4P_a}{0,162\frac{V}{T_R}} = \frac{25\,P_a T_R}{V}$

$I = \frac{P_a}{\Omega_z r^2} = \frac{P_a}{4\pi r^2}$

$\frac{25\,P_a T_R}{V} = \frac{P_a}{4\pi r^2}$

$\frac{25\,T_R}{V} = \frac{1}{4\pi r^2}$

otvoreni prostor: $\boxed{\Omega_z = 4\pi}$

$4\pi r^2 \cdot 25\,T_R = V$

$V = 100\,\pi r^2 T_R$

$\boxed{r = \sqrt{\frac{V}{100\pi T_R}}}$ => $\boxed{r = \frac{1}{10}\sqrt{\frac{V}{\pi T_R}}}$

$r = 0,1\cdot\sqrt{\frac{100}{\pi\cdot 4}} = 0,1\cdot\sqrt{\frac{25}{\pi}} = 0,1\cdot 5\cdot\frac{1}{\sqrt{\pi}} = \frac{0,5}{\sqrt{\pi}} = \boxed{0,282\,m}$

$\boxed{r = 0,282\,m}$

$$p = \frac{1}{r}\cdot\sqrt{\frac{\int c\,Pa}{4\pi}} = \frac{1}{0{,}282\,m}\cdot\sqrt{\frac{\frac{100}{400}\,\frac{kg}{m^3}\cdot\frac{m}{s}\cdot 5\,\frac{2}{s}}{4\pi}} =$$

$$= \frac{1}{0{,}282\,m}\cdot\sqrt{\frac{10\sqrt{5}}{\pi}\,\underbrace{\frac{kg}{m^2 s^2}\cdot kg\,\frac{m}{s^2}\cdot m}_{\frac{N^2}{m^2}}} = \frac{1}{0{,}282\,m}\cdot\frac{10\sqrt{5}}{\sqrt{\pi}}\,\frac{N}{m} =$$

$$= \frac{10}{0{,}282\,m}\cdot\sqrt{\frac{5}{\pi}}\,\frac{N}{m} = \boxed{44{,}7\,Pa} \Rightarrow \boxed{p = 44{,}7\,Pa}$$

Nivo zvučnog pritiska L [dB]:

I način: $L = 20\log\frac{p}{p_0} = 20\log\left(\frac{44{,}7\,Pa}{2\cdot 10^{-5}\,Pa}\right) = 20\log\left(\frac{44{,}7}{2}\cdot 10^{5}\right) =$

$$= 20\log\left(\frac{44{,}7}{2}\right) + 20\log 10^{5} = \underbrace{20\log\left(\frac{44{,}7}{2}\right)}_{27} + 100 = \boxed{127\,dB}$$

II ~~način~~: $L = 10\log\frac{I}{I_0} = 10\log\left(\frac{25\,Pa\,TR}{I_0 V}\right) = 10\log\left(\frac{25\cdot 5\cdot 4}{10^{-12}\cdot 10^{2}}\right) =$

$$= 10\log\left(\frac{10^{2}\cdot 5}{10^{-12}\cdot 10^{2}}\right) = 10\log\left(\frac{5}{10^{-12}}\right) = 10\log\left(5\cdot 10^{12}\right) =$$

$$= 10\log 5 + 10\log 10^{12} = 10\log 5 + 10\cdot 12\cdot \underbrace{\log 10}_{1} = \underbrace{10\log 5}_{7} + 120 =$$

$$= \boxed{127\,dB}$$

$$\Rightarrow \boxed{L = 127\,dB}$$

(4.) Tačkasti izvor buke instaliran je u sredini prostorije dimenzija 10 × 8 × 5 m, sa vremenom reverberacije 0,45. Odrediti na kom rastojanju od izvora je intenzitet direktnih zvučnih talasa jednak prosečnom intenzitetu reflektovanih zvučnih talasa. (4. vežba)

$a = 10\,m$

$b = 8\,m$

$c = 5\,m$

$T_R = 0{,}45 \Rightarrow T_R < 0{,}8\,s \Rightarrow$ Ajringov obrazac

$r_g = ?$

$S = 2(ab + ac + bc) = 2 \cdot (10 \cdot 8 + 10 \cdot 5 + 8 \cdot 5) = 2 \cdot (80 + 50 + 40) = 2 \cdot 170 = \boxed{340\,m^2}$

$\Rightarrow \boxed{S = 340\,m^2}$

$V = abc = 10 \cdot 8 \cdot 5\,m^3 = \boxed{400\,m^3} \Rightarrow \boxed{V = 400\,m^3}$

$$\boxed{T_R = 0{,}162 \cdot \frac{V}{-S \ln(1-\bar{\alpha})}}$$ Ajringov obrazac

$$-S \ln(1-\bar{\alpha})\, T_R = 0{,}162\, V$$

$$-S \ln(1-\bar{\alpha}) = 0{,}162 \frac{V}{T_R}$$

$$-\ln(1-\bar{\alpha}) = 0{,}162 \frac{V}{T_R S}$$

$$\ln \frac{1}{1-\bar{\alpha}} = 0{,}162 \frac{V}{T_R S}$$

$$\frac{1}{1-\bar{\alpha}} = e^{\frac{0{,}162\,V}{T_R S}}$$

$$1-\bar{\alpha} = e^{-\frac{0{,}162\,V}{T_R S}}$$

$$\Rightarrow \boxed{\bar{\alpha} = 1 - e^{-\frac{0{,}162\,V}{T_R S}}}$$

$$\bar{\alpha} = 1 - e^{-\frac{0{,}162 \cdot 400}{0{,}45 \cdot 340}}$$

$$\bar{\alpha} = 1 - e^{-\frac{0{,}162 \cdot 40}{0{,}4 \cdot 34}}$$

$$\boxed{\bar{\alpha} = 0{,}379} > 0{,}3$$

Pošto je $\bar{\alpha} > 0,3$ => nisu ispunjeni uslovi homogenosti i difuznosti zračnog polja u prostoriji

$$I_d = \frac{P_a}{4\pi r^2}$$

$$I_r = \frac{4P_a}{A}(1-\bar{\alpha})$$

$$\boxed{I_d = I_r} \Rightarrow (r_g)$$

$$\frac{P_a}{4\pi r_g^2} = \frac{4P_a}{A}(1-\bar{\alpha})$$

$$\frac{1}{4\pi r_g^2} = \frac{4}{A}(1-\bar{\alpha}) \Rightarrow A = 16\pi r_g^2 (1-\bar{\alpha})$$

$$r_g^2 = \frac{A}{16\pi(1-\bar{\alpha})} \Rightarrow \boxed{r_g = \sqrt{\frac{A}{16\pi(1-\bar{\alpha})}}}$$

$$\boxed{r_g = \sqrt{\frac{S\bar{\alpha}}{16\pi(1-\bar{\alpha})}}}$$

$$r_g = \sqrt{\frac{340 \cdot 0,379}{16\pi \cdot (1-0,379)}} \Rightarrow \boxed{r_g = 2,03\,m}$$

(5.) Na sredini plafona prostorije dimenzija 10 × 5 × 4 m postavljen je neusmereni izvor buke zvučne snage 4 W. Zidovi i plafon su prevučeni materijalom srednje vrednosti koeficijenta apsorpcije 0,5, a pod gumenim prevlačem koeficijenta apsorpcije 0,06. Odrediti:

(a) nivo buke na rastojanju 2 m od izvora,

(b) gustinu energije zvuka difuznog zvučnog polja u prostoriji i

(c) vreme reverberacije prostorije (Sabinjiga)

$a = 10\,m$

$b = 5\,m$

$c = 4\,m$

$P_a = 4\,W$

$\bar{\alpha}_1 = 0{,}5$

$\bar{\alpha}_2 = 0{,}06$

$S = 2(ab + ac + bc)$

$S = 2 \cdot (10 \cdot 5 + 10 \cdot 4 + 5 \cdot 4)$

$S = 2 \cdot (50 + 40 + 20) = 2 \cdot 110 = \boxed{220\,m^2}$

$\boxed{S = 220\,m^2}$

$V = abc = 10 \cdot 5 \cdot 4\,m^3 = \boxed{200\,m^3}$

$\boxed{V = 200\,m^3}$

(a) $r = 2\,m$, $L = ?$

(b) $E = ?$

(c) $T_{12} = ?$

$S_{pl.} = 10 \cdot 5 = \boxed{50\,m^2}$ – površina plafona

$S_p = S_{pl.} = \boxed{50\,m^2}$ – površina poda

$S_z = 2 \cdot (10 \cdot 4 + 5 \cdot 4) = 2 \cdot (40 + 20) = 2 \cdot 60 = \boxed{120\,m^2}$ – površina zidova

$A = \sum_i S_i \cdot \alpha_i = S_{pl} \cdot \bar{\alpha}_1 + S_p \cdot \bar{\alpha}_2 + S_z \cdot \bar{\alpha}_1$

$A = 50 \cdot 0{,}5 + 50 \cdot 0{,}06 + 120 \cdot 0{,}5 = 170 \cdot 0{,}5 + 50 \cdot 0{,}06 = 85 + 3 = \boxed{88\,m^2}$

$A = 50 \cdot 0{,}5 + 50 \cdot 0{,}06 + 120 \cdot 0{,}5 = 170 \cdot 0{,}5 + 50 \cdot 0{,}06 = 85 + 3 = \boxed{88\,m^2}$

$\boxed{A = 88\,m^2}$

$A = S\bar{\alpha} \Rightarrow \bar{\alpha} = \frac{A}{S} = \frac{88\,m^2}{220\,m^2} = \frac{88}{220} = \frac{44}{110} = \frac{22}{55} = \boxed{0,4}$

$\boxed{\bar{\alpha} = 0,4}$

$\bar{\alpha} > 0,3 \Rightarrow$ nisu ispunjeni uslovi homogenosti i difuznosti zvučnog polja u prostoriji

$I_d = \frac{P_a}{\Omega_z r^2} = \frac{P_a}{2\pi r^2}$ $\boxed{\Omega_z = 2\pi}$ na plafonu

$I_r = \frac{4P_a}{A}(1-\bar{\alpha})$

$\boxed{I_d = I_r} \Rightarrow (r_g)$

$\frac{P_a}{2\pi r_g^2} = \frac{4P_a}{A}(1-\bar{\alpha})$

$A = 2\pi r_g^2 \cdot 4(1-\bar{\alpha}) = 8\pi r_g^2 (1-\bar{\alpha})$

$r_g^2 = \frac{A}{8\pi(1-\bar{\alpha})} \Rightarrow \boxed{r_g = \sqrt{\frac{A}{8\pi(1-\bar{\alpha})}}}$

$r_g = \sqrt{\frac{88\,m^2}{8\pi(1-0,4)}} = \sqrt{\frac{11\,m^2}{0,6\pi}} \Rightarrow \boxed{r_g = 2,4\,m}$

$I = I_d + I_r = \frac{P_a}{2\pi r^2} + \frac{4P_a}{A}(1-\bar{\alpha})$

$I = P_a\left[\frac{1}{2\pi r^2} + \frac{4}{A}(1-\bar{\alpha})\right]$

$I = 4\,W\left[\frac{1}{2\pi \cdot 4m^2} + \frac{4}{88\,m^2}(1-0,4)\right]$

$I = 4W\left[\frac{1}{8\pi m^2} + \frac{1}{22m^2}\cdot 0,6\right] = 4\left(\frac{1}{8\pi} + \frac{0,6}{22}\right)\frac{W}{m^2}$

$\boxed{I = 0,268\,\frac{W}{m^2}}$ $I_0 = 10^{-12}\,\frac{W}{m^2}$

$L = 10\log\frac{I}{I_0} = 10\log\left(\frac{0,268\,W/m^2}{10^{-12}\,W/m^2}\right) = 10\log(0,268\cdot 10^{12}) = \boxed{114,3\,dB}$

(b) $I = \frac{4Pa}{A} = \frac{4 \cdot 4\,W}{88\,m^2} = \frac{4}{22}\,\frac{W}{m^2} = \frac{2}{11}\,\frac{W}{m^2} \Rightarrow \boxed{I = 0,18\,\frac{W}{m^2}}$

$E = \frac{I}{c} = \frac{0,18\,\frac{W}{m^2}}{340\,\frac{m}{s}} = \boxed{5,35 \cdot 10^{-4}\,\frac{J}{m^3}}$ $\quad [J = W \cdot s]$

(c) vrijeme reverberacije

Sabinov obrazac: $T_R = 0,162\,\frac{V}{A} = 0,162 \cdot \frac{200}{88} = \boxed{0,37\,s} < 0,85$

Ajringov obrazac (pošto je $T_R < 0,85$)

$$\boxed{T_R = 0,162\,\frac{V}{-S \ln(1-\bar{\alpha})}}$$

$$T_R = 0,162 \cdot \frac{200}{-220 \ln(1-0,4)} = 0,162\,\frac{200}{-220 \ln 0,6} =$$

$$= 0,162 \cdot \frac{20}{22(-\ln 0,6)} = 0,162 \cdot \frac{10}{11 \cdot (-\ln 0,6)} \Rightarrow \boxed{T_R = 0,29\,s}$$

(6.) U prostoriji sa difuznim zvučnim poljem radi nepoznat broj izvora buke iste zvučne snage. Nivo buke u prostoriji se uvećanjem još tri ista izvora buke povećа za 4 dB. Odrediti prvobitan broj izvora buke, ako oni emituju zvuk širokopojasnog frekvencijskog spektra. (6. knjiga)

$\Delta L = 4\,dB \qquad n \rightarrow n+3$

$n = ?$

$I = \frac{4Pa}{A}$, $\boxed{\bar{\alpha} < 0,3}$ ima dosta refleksije

$\Rightarrow$ homogeno zvučno polje

$I_i = \frac{4P_{ai}}{A}$, $P_{a1} = P_{a2} = \ldots = P_{an}$

$\Rightarrow I_1 = I_2 = \ldots = I_n = I$

$$\boxed{I_R = nI}$$

$$\boxed{I_R' = (n+3)I}$$

$$\Delta L = L_R' - L_R = 10\log\frac{I_R'}{I_0} - 10\log\frac{I_R}{I_0} = 10\left(\log\frac{I_R'}{I_0} - \log\frac{I_R}{I_0}\right) =$$

$$= 10\log\left(\frac{\frac{I_R'}{I_0}}{\frac{I_R}{I_0}}\right) = 10\log\left(\frac{I_R'}{I_R}\right) = 10\log\left(\frac{(n+3)I}{nI}\right) = \boxed{10\log\left(\frac{n+3}{n}\right)}$$

$$\Rightarrow \boxed{\Delta L = 10\log\left(\frac{n+3}{n}\right)}$$

$$\log\frac{n+3}{n} = \frac{\Delta L}{10}$$

$$\frac{n+3}{n} = 10^{\frac{\Delta L}{10}} \Rightarrow n+3 = n\cdot 10^{\frac{\Delta L}{10}}$$

$$n\cdot 10^{\frac{\Delta L}{10}} - n = 3$$

$$n\left(10^{\frac{\Delta L}{10}} - 1\right) = 3 \Rightarrow \boxed{n = \frac{3}{10^{\frac{\Delta L}{10}} - 1}}$$

$$n = \frac{3}{10^{\frac{4}{10}} - 1} = \frac{3}{10^{\frac{2}{5}} - 1} = \boxed{2} \Rightarrow \boxed{n = 2}, \quad n \in \mathbb{Z}$$